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Consistent cosmology with Higgs thermal inflation in a minimal extension of the MSSM

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Abstract. We consider a class of supersymmetric inflation models, in which minimal gauged F-term hybrid inflation is coupled renormalisably to the minimal supersymmetric standard model (MSSM), with no extra ingredients; we call this class the "minimal hybrid inflationary supersymmetric standard model" (MHISSM). The singlet inflaton couples to the Higgs as well as the waterfall fields, supplying the Higgs μ-term. We show how such models can exit inflation to a vacuum characterised by large Higgs vevs, whose vacuum energy is controlled by supersymmetry-breaking. The true ground state is reached after an intervening period of thermal inflation along the Higgs flat direction, which has important consequences for the cosmology of the F-term inflation scenario. The scalar spectral index is reduced, with a value of approximately 0.976 in the case where the inflaton potential is dominated by the 1-loop radiative corrections. The reheat temperature following thermal inflation is about 10^9 GeV, which solves the gravitino overclosure problem. A Higgs condensate reduces the cosmic string mass per unit length, rendering it compatible with the Cosmic Microwave Background constraints without tuning the inflaton coupling. With the minimal $U(1)'$ gauge symmetry in the inflation sector, where one of the waterfall fields generates a right-handed neutrino mass, we investigate the Higgs thermal inflation scenario in three popular supersymmetry-breaking schemes: AMSB, GMSB and the CMSSM, focusing on the implications for the gravitino bound. In AMSB enough gravitinos can be produced to account for the observed dark matter abundance through decays into neutralinos. In GMSB we find an upper bound on the gravitino mass of about a TeV, while in the CMSSM the thermally generated gravitinos are sub-dominant. When Big Bang Nucleosynthesis constraints are taken into account, the unstable gravitinos of AMSB and the CMSSM must have a mass O(10) TeV or greater, while in GMSB we find an upper bound on the gravitino mass of O(1) TeV.

Keywords: Supersymmetry, Higgs, inflation, cosmic strings

Contents

1 Introduction

Inflation is the accepted paradigm for the very early universe, thanks to its power to account accurately for cosmological data in one simple framework. However, it raises a number of theoretical problems, principally the identity of the inflaton, the flatness of its potential, and how it is coupled to the Standard Model.

A technically natural way of achieving a flat potential is through supersymmetry (SUSY). However, the flatness is generically spoiled in supergravity [1], which must be taken into account if the inflaton changes by an amount of order the Planck scale or more ("large-field" inflation). Given the large parameter space of supergravity

theories, this motivates starting the search for a supersymmetric theory of inflation with small-field inflation, in the context of a renormalisable theory.

At the same time, low energy supersymmetry remains an attractive theoretical framework in which to understand the smallness of the electroweak scale relative to the Planck scale. The Minimal Supersymmetric Standard Model (MSSM) is the most economical possibility to combine low energy SUSY with the phenomenological triumph of the Standard Model (although the high Higgs mass and the absence of positive results from the Tevatron and LHC increases the amount of parameter tuning required).

Indeed, the MSSM itself can realise inflation along one of the many flat directions [2] with the addition of non-renormalisable couplings. Inflation takes place near an inflection point in the potential, where trilinear and soft mass terms are balanced against each other, although the amount of tuning required [3] reduces the attractiveness of the scenario. The tuning can be reduced by extending the MSSM [4, 5].

The simplest class of renormalisable supersymmetric inflation models is minimal F-term hybrid inflation, by which we mean the first supersymmetric model of Ref. [1], characterised by the superpotential

$$W_I = \lambda_1 \Phi\overline{\Phi} S - M^2 S. \tag{1.1}$$

General theoretical considerations of small-field inflation drive one towards this model [6], which works without a Planck-scale inflaton field, non-renormalisable operators, or supersymmetry-breaking terms. It invokes an inflaton sector of (at least) 3 chiral superfields, consisting of the inflaton itself, S, and two waterfall [7] fields, $\Phi, \overline{\Phi}$, with an optional gauge superfield.[1] Because the the inflaton field appears linearly in the superpotential, it does not suffer from the generic supergravity problem of Hubble-scale mass terms during inflation [1].

In its standard form, however, F-term hybrid inflation suffers from a number of problems which reduce its power to fit cosmological data. First and foremost is the gravitino problem, which limits the reheat temperature to be unnaturally small compared with the inflation scale. Of less severity is the spectral index problem. If the inflaton potential is dominated by the 1-loop radiative corrections, F-term hybrid inflation predicts that the spectral index of cosmological perturbations N e-foldings before the end of inflation is $n_s = 1 - 1/N$. For the canonical 60 e-foldings, this is more than 1σ above the WMAP7 value $n_s = 0.963 \pm 0.012$. Finally, many models generate cosmic strings, and the CMB constraints on their mass per unit length forces one to very weak inflaton couplings, where $n_s \to 1$ [8].

There also remains the question of how the inflaton sector is coupled to the MSSM. If we restrict ourselves to renormalisable theories combining minimal U(1)$'$-gauged F-term hybrid inflation with the MSSM, with no other fields, and preserving all the symmetries, the choices are limited. The singlet inflaton S can couple in the superpotential only to the product of the Higgs fields or the square of the right-handed neutrino fields (which we take to be included the MSSM). If the MSSM fields have

[1]The number of chiral superfields can be reduced to 2 without a gauge field, or if they are in a real representation.

non-trivial charge assignments under the U(1)′ of F-term inflation, the coupling of S to the neutrinos is forbidden, and its place taken by one of the waterfall fields. This has the nice feature of generating a see-saw mechanism, with the neutrino masses also controlled by the vev of the waterfall fields. Neutrino masses are also allowed if the waterfall fields are U(2) triplets, with $SU(2)_R$ as a subgroup.

We will refer to the minimal case where the symmetry of the waterfall fields is U(1)′ as the Minimal Hybrid Inflationary Supersymmetric Standard Model (MHISSM). In the model, it is very natural that the gauge singlet inflaton S should be coupled both to the waterfall fields and to the Higgs fields, which mixes the standard MSSM Higgs flat direction with the hybrid inflation waterfall direction. If the coupling of the inflaton to the Higgs is smaller than to the waterfall fields, inflation ends with the development of vevs for the Higgs multiplets, $h_{1,2}$, breaking the electroweak symmetry. Soft terms lift the flat direction, and if certain constraints are satisfied, the Higgs fields will finally reach the standard vacuum after a period of thermal inflation, with a reheat temperature of about 10^9 GeV. This solves the gravitino overclosure problem, and Big Bang Nucleosynthesis constraints can be satisfied with massive (O(10) TeV or more) or stable gravitinos [9–12].

We call this second period of accelerated expansion Higgs thermal inflation. It is a natural consequence of the coupling of the F-term hybrid inflaton to the Higgs fields, and offers a generic solution to the gravitino problem. At the same time, a TeV-scale vacuum expectation value for the inflaton generates an effective μ-term. The model was first introduced in Ref. [13] in the context of Anomaly-Mediated Supersymmetry Breaking (AMSB). We termed the version of AMSB there deployed *strictly* anomaly mediated supersymmetry breaking (sAMSB), because D-terms associated with the U(1)′ symmetry resolve the AMSB tachyonic slepton problem, without requiring an additional explicit source of supersymmetry breaking.

In this paper we demonstrate that the interesting cosmological consequences, in particular Higgs thermal inflation, are a result of the structure of the model at the inflation scale, and not of the particular supersymmetry-breaking scenario. We derive the effective potential for the combination of fields driving thermal inflation, and the constraints on the soft breaking parameters for a phenomenologically acceptable ground state, in three popular supersymmetry-breaking scenarios: anomaly-mediated (AMSB), gauge-mediated (GMSB) and the constrained minimal supersymmetric standard model (CMSSM). We find that the lower reheat temperature following thermal inflation solves the gravitino problem in the CMSSM, while in AMSB enough gravitinos can be produced to account for the observed dark matter abundance through decays into neutralinos. In GMSB we find an upper bound on the gravitino mass of about a TeV, derived from constraints on NLSP decays during and after Big Bang Nucleosynthesis (BBN).

F-term models with Higgs thermal inflation have other important features. The spectral index of scalar Cosmic Microwave Background fluctuations n_s is reduced, as fewer e-foldings of F-term inflation are required. In the range of couplings for which the 1-loop radiative corrections dominate the inflaton potential, we find $n_s = 0.976(1)$, where the uncertainty comes from the spread of reheat temperatures in that range. The cosmic string mass per unit length is greatly reduced by the presence of a Higgs

condensate at the string core, and is rendered independent of the inflaton coupling. Finally, thermal inflation sweeps away the gravitinos generated at the first stage of inflation, and any GUT-scale relics such as magnetic monopoles.

There are other models which renormalisably couple F-term hybrid inflation to the MSSM. $\mathrm{F_D}$ hybrid inflation [14, 15] has the same field content as ours, but the MSSM has no $U(1)'$ charges; and it requires a Fayet Iliopoulos term. Also potentially in the class is the B−L model of Refs. [16–18], although there is no explicit discussion of the coupling of the inflaton to the Higgs fields. In the model of Ref. [19] the waterfall fields are $SU(2)_R$ triplets. The authors identified a flat direction involving the Higgs, without pursuing its consequences. The original F-term inflation model [1] had a spontaneously broken global U(1) symmetry, and models based on coupling it to the MSSM have recently been explored in [20], again without the possibility of Higgs thermal inflation being noticed. The same field content can also produce a promising superconformal D-term inflation model [21].

Further afield, it is also possible to construct renormalisable models of inflation in the Next-to-Minimal Supersymmetric Standard Model using soft terms to generate the vacuum energy [22]. Inflation along a flat direction which mixes a singlet with an MSSM flat direction has also been investigated recently in Ref. [23]. In that work, a single stage of inflation was envisaged, and in order to supply a satisfactory spectral index, the coupling to the inflaton has to be non-renormalisable.

The spectral index problem can also be solved with a non-minimal Kähler potential [24], or tuning the inflaton coupling to be small enough that the linear soft term dominates its potential [25]. In this paper we will restrict ourselves to the case where radiative corrections dominate the inflaton potential, and the Kähler potential is canonical.

2 Coupling F-term inflation and the MSSM

Our guiding principle is to couple minimal F-term hybrid inflation and the MSSM (which we take to include 3 families of right-handed neutrinos) in a renormalisable way, preserving all symmetries including supersymmetry (while allowing soft breaking terms in both sectors). Hence the superpotential will take the form

$$W = W_I + W_A + W_X \tag{2.1}$$

where W_I is the standard linear F-term hybrid inflation superpotential of Eq. (1.1), W_A is the MSSM Yukawa superpotential

$$W_A = H_2QY_UU + H_1QY_DD + H_1LY_EE + H_2LY_NN, \tag{2.2}$$

and W_X is the coupling between the inflaton sector and the MSSM superpotential, containing renormalisable terms only. We will assume that the $U(1)'$ symmetry of the waterfall fields

$$\Phi \to \Phi' = e^{iq_\Phi\theta}\Phi, \quad \overline{\Phi} \to \overline{\Phi}' = e^{iq_{\overline{\Phi}}\theta}\overline{\Phi} \tag{2.3}$$

	Q	U	D	H_1	H_2	N
q	$-\frac{1}{3}q_L$	$-q_E-\frac{2}{3}q_L$	$q_E+\frac{4}{3}q_L$	$-q_E-q_L$	q_E+q_L	$-2q_L-q_E$

Table 1. Anomaly free U(1) charges for lepton doublet, singlet charges q_L, q_E respectively.

is gauged. The inflaton S must be a gauge singlet, and so $q_{\overline{\Phi}} = -q_\Phi$. The mass scale M sets the inflation scale and the vevs of Φ and $\overline{\Phi}$. Given that the inflation scale is of order 10^{14} GeV, the waterfall fields must be SU(3)⊗SU(2)⊗U(1)$_Y$ singlets. Note that W_I has a global U(1) R-symmetry, which forbids the terms S^2, S^3 and $\Phi\overline{\Phi}$. In order to preserve the flat potential for the inflaton, we must preserve this symmetry; we will discuss more of its implications in a moment.

The form of W_X is now tightly constrained by symmetry and anomaly cancellation. Possible anomaly-free U(1)$'$ charge assignments for the MSSM fields are shown in Table 1. The SM gauged U(1)$_Y$ is $q_L = -1, q_E = 2$. U(1)$_{B-L}$ is $q_E = -q_L = 1$; in the absence of N this would have $\mathrm{U(1)}^3$ and U(1)-gravitational anomalies. The diagonal subgroup of SU(2)$_R$ is $q_L = 0, q_E = 1$. Note that quite generally $q_{H_1} = -q_{H_2}$, so we will write $q_{H_2} = -q_{H_1} = q_H$. We will assume that the MSSM fields couple to a U(1)$'$ distinct from U(1)$_Y$, i.e. that $2q_L+q_E \neq 0$, and moreover that in the AMSB case the values of q_L and q_E result in a solution to the AMSB tachyonic slepton problem [26]. For the resulting sparticle spectra in this case, see Ref. [13]. (Note that if the U(1)$'$ does not couple to MSSM fields, we are driven to $\mathrm{F_D}$ inflation [14, 15]). Three SU(3)⊗SU(2)⊗U(1)$_Y$ singlets quadratic in the MSSM fields are available for W_X, namely H_1H_2, LH_2 and NN [27]. The U(1)$'$ charge assignments, combined with the global R-symmetry, with superfield charges

$$S = 2, L = E = N = U = D = Q = 1, H_1 = H_2 = \Phi = \overline{\Phi} = 0, \tag{2.4}$$

now uniquely specify the coupling term as

$$W_X = \tfrac{1}{2}\lambda_2 NN\Phi - \lambda_3 SH_1H_2, \tag{2.5}$$

where we have set $q_{\Phi,\overline{\Phi}} = \pm(4q_L + 2q_E)$ to permit the first term. All renormalisable B, L violating interactions and the NN and LH_2 mass terms are forbidden by the U(1)$'$ gauge invariance, and the superpotential Eq. (2.1) contains all renormalisable terms consistent with U(1)$'$ and the R-symmetry. Note in particular that the R-symmetry forbids the Higgs μ-term H_1H_2. Moreover, the R-symmetry forbids the quartic superpotential terms $QQQL$ and $UUDE$, which are allowed by the U(1)$'$ symmetry, and give rise to dimension 5 operators capable of causing proton decay [28, 29]. In fact the charges in Eq. (2.4) disallow B-violating operators in the superpotential of arbitrary dimension.

Soft terms break the continuous R-symmetry to the usual R-parity. The lightest supersymmetric particle (LSP) is therefore stable. (From Eq. (2.4), the LSP is a scalar quark or lepton, or a gaugino, or a fermionic Higgs, S, Φ or $\overline{\Phi}$.)

To summarise the assumptions which force us to this unique class of theories, we require a theory with :

1. The field content of minimal F-term inflation and the MSSM.
2. The symmetries of minimal F-term inflation and the MSSM.
3. Renormalisable couplings only.
4. An inflaton-sector $U(1)'$ gauge symmetry which is coupled to the MSSM.

Note that if Φ and $\overline{\Phi}$ are gauged under a larger symmetry group, the coupling $NN\Phi$ is not allowed, unless they are triplets of $SU(2)_R$ and (N, E) are doublets [19].

The parameters M, λ_1, λ_3 are real and positive and λ_2 is a symmetric 3×3 matrix which we will take to be real and diagonal. The sign of the λ_3 term above is chosen because with our conventions, in the electroweak vacuum

$$H_1 = \left(\frac{v_1}{\sqrt{2}}, 0\right)^T \quad \text{and} \quad H_2 = \left(0, \frac{v_2}{\sqrt{2}}\right)^T \tag{2.6}$$

we have $H_1 H_2 \to -\frac{1}{2} v_1 v_2$.

In the following we will denote the $SU(3)\otimes SU(2)\otimes U(1)_Y$ gauge couplings by g_3, g_2 and g_1, and the $U(1)'$ gauge coupling by g'. The normalisation of the $U(1)_Y$ gauge coupling corresponds to the usual SM convention, not that appropriate for SU(5) unification. We will denote the soft parameters for the gaugino masses M_a, for a cubic interaction with Yukawa coupling λ h_λ, and for a mass term $\phi^*\phi$ (where ϕ denotes a scalar field), m_ϕ^2. For the one mass term of the form ϕ^2 in the MSSM $(H_1 H_2)$ we will use m_3^2.

3 The Higgs potential and its extrema

In this section we explore the important extrema of the Higgs potential, and demonstrate that there is a 1-parameter family of supersymmetric ground states with non-zero vevs for $\phi, \overline{\phi}$ and $h_{1,2}$ before supersymmetry-breaking is taken into account. We will assume that M, the scale of inflation and $U(1)'$ symmetry-breaking, is much larger than the scale of supersymmetry-breaking.

The existence of the one-parameter family (before thermal effects and soft terms are taken into account), is demonstrated as follows. The minimum of the scalar potential is determined by the requirement that both the F- and D-terms vanish. The vanishing of the D-terms ensures that $|\phi| = |\overline{\phi}|$, $|h_1| = |h_2|$ and $h_1^\dagger h_2 = 0$, while the vanishing of the F-term is assured by $\lambda_1 \phi\overline{\phi} - \lambda_3 h_1 h_2 = M^2$. The minimum can therefore be parametrised by an SU(2) gauge transformation and angles χ, φ defined by

$$\begin{aligned} \langle h_1 \rangle &\simeq i\sigma_2 \langle h_2 \rangle^* \simeq \left(\frac{M}{\sqrt{\lambda_3}} \cos\chi, 0\right), \\ \langle \phi \rangle &\simeq \langle \overline{\phi}^* \rangle \simeq \frac{M}{\sqrt{\lambda_1}} \sin\chi e^{i\varphi}. \end{aligned} \tag{3.1}$$

The φ angle can always be removed by a $\mathrm{U}(1)''$ gauge transformation (where the residual symmetry unbroken by the Higgs vevs alone is $\mathrm{U}(1)_{\rm em} \times \mathrm{U}(1)''$), so the physical flat direction just maps out the interval $0 \leq \chi \leq \pi/2$. At the special point $\chi = 0$ the $\mathrm{U}(1)''$ symmetry is restored, and at $\chi = \pi/2$ the $\mathrm{SU}(2) \otimes \mathrm{U}(1)_Y$ is restored. Away from these special points only $\mathrm{U}(1)_{\rm em}$ is unbroken.

The degenerate minima have been noted before [19] in a model with gauge group $\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$. However, the important cosmological consequences which follow was first explored in Ref. [13].

Let us first consider the limiting cases where either $h_{1,2}$ or $\phi, \overline{\phi}$ vanish.

3.1 The $\phi, \overline{\phi}, s$ extremum (ϕ-vacuum)

In the $\phi, \overline{\phi}, s$ subspace (lower case fields denote the scalar component of the superfields) the scalar potential (including soft supersymmetry-breaking terms) is:

$$\begin{aligned} V =\ & \lambda_1^2(|\phi s|^2 + |\overline{\phi} s|^2) + |\lambda_1 \phi\overline{\phi} - M^2|^2 + \tfrac{1}{2} q_\Phi^2 g'^2 \left(|\phi|^2 - |\overline{\phi}|^2\right)^2 \\ & + m_\phi^2 |\phi|^2 + m_{\overline{\phi}}^2 |\overline{\phi}|^2 + m_s^2 |s|^2 + \rho M^2 m_{\frac{3}{2}} (s + s^*) \\ & + h_{\lambda_1} \phi\overline{\phi} s + c.c.. \end{aligned} \tag{3.2}$$

We will assume that the term linear in s is small enough not to be important for inflation (and quantify this smallness in Section 5). In AMSB there are arguments [30] to show that, without a quadratic term S^2 in the superpotential, the only RG invariant solution for ρ is $\rho = 0$.

Let us establish the minimum in this subspace, under the assumption that $m_{\frac{3}{2}} \ll M$. We shall call this the ϕ-vacuum. With the notation $\langle \phi \rangle = v_\phi/\sqrt{2}$, $\langle \overline{\phi} \rangle = v_{\overline{\phi}}/\sqrt{2}$ and $\langle s \rangle = v_s/\sqrt{2}$, we find

$$v_\phi \left[m_\phi^2 + \tfrac{1}{2}\lambda_1^2 v_s^2 + \tfrac{1}{2} g^2 q_\Phi^2 (v_\phi^2 - v_{\overline{\phi}}^2) \right] + v_{\overline{\phi}} \left[\lambda_1 \left(\tfrac{1}{2}\lambda_1 v_\phi v_{\overline{\phi}} - M^2 \right) + \frac{h_{\lambda_1}}{\sqrt{2}} v_s \right] = 0, \tag{3.3}$$

$$v_{\overline{\phi}} \left[m_{\overline{\phi}}^2 + \tfrac{1}{2}\lambda_1^2 v_s^2 - \tfrac{1}{2} g^2 q_\Phi^2 (v_\phi^2 - v_{\overline{\phi}}^2) \right] + v_\phi \left[\lambda_1 \left(\tfrac{1}{2}\lambda_1 v_\phi v_{\overline{\phi}} - M^2 \right) + \frac{h_{\lambda_1}}{\sqrt{2}} v_s \right] = 0, \tag{3.4}$$

$$v_s \left[m_s^2 + \tfrac{1}{2}\lambda_1^2 (v_\phi^2 + v_{\overline{\phi}}^2) \right] + \frac{h_{\lambda_1}}{\sqrt{2}} v_\phi v_{\overline{\phi}} + \sqrt{2}\rho M^2 m_{\frac{3}{2}} = 0. \tag{3.5}$$

From Eqs. (3.3), (3.4) we find

$$\lambda_1 \left(\tfrac{1}{2}\lambda_1 v_\phi v_{\overline{\phi}} - M^2 \right) = -\frac{v_\phi v_{\overline{\phi}}}{v_\phi^2 + v_{\overline{\phi}}^2} \left[m_\phi^2 + m_{\overline{\phi}}^2 + \lambda_1^2 v_s^2 \right] - \frac{h_{\lambda_1}}{\sqrt{2}} v_s, \tag{3.6}$$

$$\tfrac{1}{2} g'^2 q_\Phi^2 (v_\phi^2 - v_{\overline{\phi}}^2) = \frac{v_{\overline{\phi}}^2 m_{\overline{\phi}}^2 - v_\phi^2 m_\phi^2 + (v_{\overline{\phi}}^2 - v_\phi^2)\tfrac{1}{2}\lambda_1^2 v_s^2}{v_\phi^2 + v_{\overline{\phi}}^2}. \tag{3.7}$$

Then from Eqs. (3.6), (3.7), to leading order in an expansion in $m_{\frac{3}{2}}/M$ we have

$$v_\phi^2 \simeq v_{\overline{\phi}}^2 \simeq \frac{2}{\lambda_1} M^2, \tag{3.8}$$

and from Eq. (3.5) that v_s is $O(m_{\frac{3}{2}})$. It follows from Eq. (3.7) that

$$v_\phi^2 - v_{\bar\phi}^2 = \frac{m_{\bar\phi}^2 - m_\phi^2}{g'^2 q_\Phi^2} + O(m_{\frac{3}{2}}^4/M^2), \tag{3.9}$$

and from Eq. (3.5) that

$$v_s = -\frac{h_{\lambda_1}}{\sqrt{2}\lambda_1^2} - \frac{m_{\frac{3}{2}}\rho}{\sqrt{2}\lambda_1} + O(m_{\frac{3}{2}}^2/M). \tag{3.10}$$

From now on we neglect ρ, assuming that

$$|\rho| \lesssim \left| \frac{h_{\lambda_1}}{\lambda_1 m_{\frac{3}{2}}} \right|. \tag{3.11}$$

Substituting back from Eqs. (3.8), (3.10) into Eq. (3.2), we obtain to leading order

$$V_\phi = \frac{1}{\lambda_1} M^2 \left(m_\phi^2 + m_{\bar\phi}^2 - \frac{h_{\lambda_1}^2}{2\lambda_1^2} \right) \tag{3.12}$$

and from Eq. (3.10) a Higgs μ-term

$$\mu_h = \frac{\lambda_3 h_{\lambda_1}}{2\lambda_1^2}, \tag{3.13}$$

naturally of the same order as the supersymmetry-breaking scale.

The theory is approximately supersymmetric at the scale M, so the $\mathrm{U}(1)'$ gauge boson, the Higgs boson, the gaugino and one combination of $\psi_{\phi,\bar\phi}$ form a massive supermultiplet with mass $m \sim g'\sqrt{v_\phi^2 + v_{\bar\phi}^2}$, while the remaining combination of ϕ and $\bar\phi$ and the other combination of $\psi_{\phi,\bar\phi}$ form a massive chiral supermultiplet, with mass $m \sim \lambda_1\sqrt{v_\phi^2 + v_{\bar\phi}^2}$.[2]

The large vev for ϕ generates inflation-scale masses for the N triplet, thus naturally implementing the see-saw mechanism.

3.2 The $h_{1,2}, s$ extremum (h-vacuum)

. In the $h_{1,2}, s$ subspace, the scalar potential is

$$\begin{aligned} V = {} & \lambda_3^2(|h_1 s|^2 + |h_2 s|^2) + |\lambda_3 h_1 h_2 - M^2|^2 + \tfrac{1}{2} g'^2 q_H^2 \left(|h_1|^2 - |h_2|^2\right)^2 \\ & + \tfrac{1}{8} g_1^2 (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \tfrac{1}{8} g_2^2 \sum_a (h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2)^2 \\ & + m_{h_1}^2 |h_1|^2 + m_{h_2}^2 |h_2|^2 + m_s^2 |s|^2 + \rho M^2 m_{\frac{3}{2}} (s + s^*) \\ & + h_{\lambda_3} h_1 h_2 s + c.c.. \end{aligned} \tag{3.14}$$

[2] A detailed explanation of the symmetry-breaking is contained in Ref. [18].

Note that we assume there is no $h_1 h_2$ mass term; its absence follows from the absence of the corresponding term in the superpotential (which is forbidden by the R-symmetry) when the source of supersymmetry breaking can be represented by a non-zero vev for a spurion (or conformal compensator) field.

The structure is similar to Eq. (3.2), with the addition of SU(2) and $U(1)_Y$ D-terms. Without loss of generality the SU(2) D-term vanishes with the choice $h_1 = (v_1/\sqrt{2}, 0)$ and $h_2 = (0, v_2/\sqrt{2})$, and $v_1 = v_2$. The values of the fields at the minimum (which we term the h-vacuum) and the value of the potential at this extremum can then be recovered from the result of the previous section with the replacement $\lambda_1 \to \lambda_3$), leading to a potential energy density

$$V_h = \frac{M^2}{\lambda_3}\left(m_{h_1}^2 + m_{h_2}^2 - \frac{h_{\lambda_3}^2}{2\lambda_3^2}\right). \tag{3.15}$$

3.3 Potential along the ϕ, $\overline{\phi}$, h_1, h_2 flat direction

As we outlined at the beginning of the section, the supersymmetric minima are parametrised by an angle χ, defined in (3.1). Soft terms lift this degeneracy, and the leading terms in the effective potential for χ can be found in an expansion in $m_{\frac{3}{2}}^2/M^2$. After solving for s, it is found that

$$V(\chi) \simeq -\frac{M^2}{2}\frac{\left(\tilde{h}_{\lambda_1}\sin^2\chi + \tilde{h}_{\lambda_3}\cos^2\chi\right)^2}{\lambda_1\sin^2\chi + \lambda_3\cos^2\chi} + M^2\left(\frac{\bar{m}_\phi^2}{\lambda_1}\sin^2\chi + \frac{\bar{m}_h^2}{\lambda_3}\cos^2\chi\right), \tag{3.16}$$

where we have defined

$$\tilde{h}_{\lambda_1} = \frac{h_{\lambda_1}}{\lambda_1}, \quad \tilde{h}_{\lambda_3} = \frac{h_{\lambda_3}}{\lambda_3}, \quad \bar{m}_\phi^2 = m_\phi^2 + m_{\bar{\phi}}^2, \quad \bar{m}_h^2 = m_{h_1}^2 + m_{h_2}^2. \tag{3.17}$$

4 Supersymmetry-breaking and the true minimum

In this section we investigate under which conditions the phenomenologically acceptable large-ϕ solution is the true minimum, in three popular supersymmetry-breaking scenarios. Hence we are looking for constraints on the soft supersymmetry-breaking parameters such that

$$V_h - V_\phi = M^2\left(\frac{\tilde{h}_{\lambda_1}^2}{2\lambda_1} - \frac{\tilde{h}_{\lambda_3}^2}{2\lambda_3} - \frac{\bar{m}_\phi^2}{\lambda_1} + \frac{\bar{m}_h^2}{\lambda_3}\right) > 0, \tag{4.1}$$

$$V''(\pi/2) = \frac{2M^2}{\lambda_1}\left[-\frac{\tilde{h}_{\lambda_1}^2}{2}\left(2\frac{\tilde{h}_{\lambda_3}}{\tilde{h}_{\lambda_1}} - \frac{\lambda_3}{\lambda_1} - 1\right) + \bar{m}_h^2\frac{\lambda_1}{\lambda_3} - \bar{m}_\phi^2\right] > 0. \tag{4.2}$$

We will also check that the false vacuum at $\chi = 0$ is a local maximum, from the sign of $V''(0)$, which can be recovered from $V''(\pi/2)$ by the replacements $1 \leftrightarrow 3$ and $\bar{m}_\phi^2 \leftrightarrow \bar{m}_h^2$. A metastable false vacuum, as we will demonstrate in Section 7, would lead to the universe remaining trapped in an inflating phase.

We assume that the $\mathrm{U}(1)'$ symmetry is broken by a vev of order $v' \sim M/\sqrt{\lambda_{1,3}}$, and evaluate the soft terms at this scale, rather than running down to the electroweak scale. This is the appropriate renormalisation scale to investigate a potential with vevs of order v', whose important radiative corrections are from particles of mass of order $g'v'$ and M. Note that in inflation models, with inflaton couplings λ_1 and λ_3 are generally small, and so the $\mathrm{U}(1)'$ gauge boson mass $m_A = g'\sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$ is much greater than M, unless g' is also small.

4.1 Anomaly-mediated supersymmetry-breaking

With anomaly mediation, the soft breaking parameters take the generic renormalisation group invariant form

$$M_a = m_{\frac{3}{2}} \beta_{g_a}/g_a, \tag{4.3}$$

$$h_{U,D,E,N} = -m_{\frac{3}{2}} \beta_{Y_{U,D,E,N}}, \tag{4.4}$$

$$(m^2)^i{}_j = \frac{1}{2} m_{\frac{3}{2}}^2 \mu \frac{d}{d\mu} \gamma^i{}_j + k Y_i' \delta^i{}_j, \tag{4.5}$$

$$m_3^2 = \kappa m_{\frac{3}{2}} \mu_h - m_{\frac{3}{2}} \beta_{\mu_h}. \tag{4.6}$$

Here μ is the renormalisation scale, and $m_{\frac{3}{2}}$ is the gravitino mass; β_{g_a} are the gauge β-functions and γ is the chiral supermultiplet anomalous dimension matrix. $Y_{U,D,E,N}$ are the 3×3 Yukawa matrices, μ_h is the superpotential Higgs μ-term, κ and k are constants, and Y_i' are charges corresponding to the $\mathrm{U}(1)'$ symmetry.

In the MSSM, κ is an arbitrary parameter, which in practice is fixed by minimising the Higgs potential at the electroweak scale. The parameter k is generated by the breaking of the $\mathrm{U}(1)'$ symmetry at a large scale, and forms the basis of the solution to the tachyonic slepton problem within the framework of AMSB, as explained in [13], whence the name strictly anomaly-mediated supersymmetry-breaking (sAMSB) originates.

The Higgs μ-term, μ_h, is generated by the the vev of the inflation s, which in turn is triggered by the $\mathrm{U}(1)'$ symmetry-breaking. Hence the parameter k, and the equation for m_3^2, are relevant only below the $\mathrm{U}(1)'$ symmetry-breaking scale v'.

As a first approximation, we will assume that the g' terms dominate throughout, as q_H and q_Φ are generally large, in which case the h_{λ_1} and h_{λ_3} trilinear soft terms are given from Eq. (4.4) as:

$$h_{\lambda_1} \simeq m_{\frac{3}{2}} \frac{\lambda_1}{16\pi^2} \left(4 q_\Phi^2 g'^2\right), \tag{4.7}$$

$$h_{\lambda_3} \simeq m_{\frac{3}{2}} \frac{\lambda_3}{16\pi^2} (4 q_H^2 g'^2), \tag{4.8}$$

while the mass soft terms are

$$m_\phi^2 \simeq -m_{\frac{3}{2}}^2 \frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(2g'^2 q_\Phi^2\right), \tag{4.9}$$

$$m_{\overline{\phi}}^2 \simeq -m_{\frac{3}{2}}^2 \frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(2g'^2 q_\Phi^2\right), \tag{4.10}$$

$$m_{h_1}^2 \simeq -m_{\frac{3}{2}}^2 \frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(2g'^2 q_H^2\right), \tag{4.11}$$

$$m_{h_2}^2 \simeq -m_{\frac{3}{2}}^2 \frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(2g'^2 q_H^2\right). \tag{4.12}$$

The one loop g' β-function is

$$\beta_{g'} = Q\frac{g'^3}{16\pi^2} \tag{4.13}$$

where

$$\begin{aligned} Q &= n_G(\tfrac{40}{3}q_L^2 + 8q_E^2 + 16q_E q_L) + 36q_L^2 + 40q_E q_L + 12q_E^2 \\ &= 76q_L^2 + 36q_E^2 + 88q_E q_L \end{aligned} \tag{4.14}$$

for $n_G = 3$. Hence

$$m_\phi^2 \simeq m_{\overline{\phi}}^2 \simeq -2m_{\frac{3}{2}}^2\left(\frac{g'^2}{16\pi^2}\right)^2 q_\Phi^2 Q, \tag{4.15}$$

$$m_{h_1}^2 \simeq m_{h_2}^2 \simeq -2m_{\frac{3}{2}}^2\left(\frac{g'^2}{16\pi^2}\right)^2 q_H^2 Q. \tag{4.16}$$

Thus the difference in the energy densities between the two vacua is, in this approximation,

$$V_h - V_\phi \simeq M^2\left(\frac{m_{\frac{3}{2}}g'^2}{16\pi^2}\right)^2\left[\frac{4Qq_\Phi^2 + 8q_\Phi^4}{\lambda_1} - \frac{4Qq_H^2 + 8q_H^4}{\lambda_3}\right]. \tag{4.17}$$

The coefficient Q is in general large, and larger than both q_Φ^2 and q_H^2, so the condition for V_ϕ to be the true minimum may be written

$$\frac{\lambda_3}{\lambda_1} \gtrsim \left(\frac{q_H}{q_\Phi}\right)^2. \tag{4.18}$$

It is not hard to check from Eq. (4.2)) that under the same assumptions, the ϕ-vacuum is a minimum and the h-vacuum is a maximum. Hence no further constraints on the parameters are generated.

In the next section we will see that if $\lambda_3 > \lambda_1$, then inflation ends with $\phi, \overline{\phi}$ developing non-zero vevs, whereas if $\lambda_3 < \lambda_1$ it is $\langle h_{1,2}\rangle$ which become non-zero; this statement is independent of the nature of the soft breaking terms. Now is easy to show that $\left(\frac{q_H}{q_\Phi}\right)^2 < 1$ unless

$$-\frac{3}{5} \le \frac{q_L}{q_E} \le -\frac{1}{3}. \tag{4.19}$$

However, the domain defined by Eq. (4.19) does not permit a satisfactory electroweak vacuum in the AMSB case [26]. For example, for the specific choice $q_L = 0$, which *can* lead to an acceptable electro-weak vacuum [13], the condition $V_h > V_\phi$ becomes (from Eq. (4.17))

$$\frac{\lambda_3}{\lambda_1} \gtrsim \frac{19}{88}. \tag{4.20}$$

or $\lambda_1 \lesssim 4\lambda_3$ from the approximation Eq. (4.18).

We see, therefore, that there will generally be a domain

$$\lambda_1 \left(\frac{q_H}{q_\Phi}\right)^2 \lesssim \lambda_3 < \lambda_1 \tag{4.21}$$

such that the universe exits to the false high Higgs vev h-vacuum, evolving subsequently to the true vacuum as we shall describe later.

In the Appendix we include a more accurate computation of the vacuum energy difference, taking into account the SM gauge couplings and the top Yukawa coupling.

4.2 Gauge-mediated supersymmetry-breaking

In the GMSB framework (see e.g. [31]), supersymmetry-breaking is communicated by a set of messenger fields C which have SM gauge charges in a vector-like representation, which should be complete GUT multiplets if gauge unification is to be preserved. The messenger fields are supposed to have a large mass, given by the vev of the scalar component of a chiral superfield X, which also has a non-zero F-term F_X, the source of the supersymmetry breaking. Although there are many possibly choices for the field representations of the messenger fields, we can adapt the simple model described in [31] to study our model.

We introduce the following superpotential for the extra fields

$$W_{gm} = \lambda_4 SC\bar{C} + \lambda_5 XC\bar{C}, \tag{4.22}$$

assuming that some extra dynamics at a higher scale gives both the scalar component of X and F_X a vev. We will assume that $\langle X \rangle \gg M$. Radiative corrections from the messenger particles then induce masses for the gauginos at one loop,

$$M_a = \frac{g_a^2}{16\pi^2}\Lambda_{\rm g}, \tag{4.23}$$

where $\Lambda_{\rm g} = N_{\rm mi}\langle F_X\rangle / M_X$, $M_X = \lambda_5 \langle X \rangle$, and $N_{\rm mi}$ is the messenger index, equal to twice the sum of the Dynkin indices of the messenger fields. Scalars acquire masses from 2-loop corrections of

$$m_i^2 = 2\Lambda_{\rm s}^2 \sum_a \left(\frac{g_a^2}{16\pi^2}\right)^2 C_a(i), \tag{4.24}$$

where $\Lambda_{\rm s}^2 = N_{\rm mi}(\langle F_X\rangle / M_X)^2$, $C_a(i)$ is the quadratic Casimir associated with the ath gauge group for the ith scalar, and the sum over a includes the four gauge couplings $g_{1\to 3}, g'$.

Trilinear terms are also induced at 2 loops, and so are of order $\Lambda_g(\alpha_a/4\pi)^2$. They are small compared with the gaugino masses, and it is a reasonable approximation to take them to vanish at the messenger scale M_X. We assume that $\Lambda_{g,s}$ are of the correct order of magnitude for supersymmetry-breaking.

We thus have

$$\begin{aligned}
m_\phi^2 &= m_{\bar\phi}^2 = 2\Lambda_s^2\left(\frac{g'^2}{16\pi^2}\right)^2 q_\Phi^2, \\
m_{h_1}^2 &= m_{h_2}^2 = 2\Lambda_s^2\left[\frac{3}{4}\left(\frac{g_2^2}{16\pi^2}\right)^2 + \frac{1}{4}\left(\frac{g_1^2}{16\pi^2}\right)^2 + \left(\frac{g'^2}{16\pi^2}\right)^2 q_H^2\right], \\
\frac{h_{\lambda_1}}{\lambda_1} &= \frac{h_{\lambda_3}}{\lambda_3} = 0.
\end{aligned} \tag{4.25}$$

Thus the difference between the vacuum energies is

$$\begin{aligned}
V_h - V_\phi &= M^2\left(\frac{m_{h_1}^2 + m_{h_2}^2}{\lambda_3} - \frac{m_\phi^2 + m_{\bar\phi}^2}{\lambda_1}\right) \\
&= \frac{2\Lambda_s^2 M^2}{(16\pi^2)^2}\left[\left(\frac{3}{2}g_2^4 + \frac{1}{2}g_1^4 + 2q_H^2 g'^4\right)\frac{1}{\lambda_3} - \frac{2q_\Phi^2 g'^4}{\lambda_1}\right],
\end{aligned} \tag{4.26}$$

so that, if we assume dominance of the g' terms, the condition that $V_\phi < V_h$ becomes

$$\frac{\lambda_3}{\lambda_1} \lesssim \left(\frac{q_H}{q_\Phi}\right)^2. \tag{4.27}$$

This is precisely the opposite condition to that in AMSB, Eq. (4.18). As in AMSB, the condition that $V_\phi < V_h$ is sufficient to ensure that V_ϕ is a minimum and V_h a maximum.

Now in GMSB, we do not have the constraint on the domain (q_L, q_E) that we described in the AMSB case. Inflation will end in the Higgs phase unless

$$\left(\frac{q_H}{q_\Phi}\right)^2 > 1 \quad \text{and} \quad 1 < \frac{\lambda_3}{\lambda_1} < \left(\frac{q_H}{q_\Phi}\right)^2, \tag{4.28}$$

in which case it ends directly in the true ϕ-vacuum.

4.3 Constrained minimal supersymmetric standard model

At the high scale we will have the CMSSM pattern of soft breaking parameters,

$$\begin{aligned}
m_\phi^2 &= m_{\bar\phi}^2 = m_{h_1}^2 = m_{h_2}^2 = m_0^2, \\
\frac{h_{\lambda_1}}{\lambda_1} &= \frac{h_{\lambda_3}}{\lambda_3} = A
\end{aligned} \tag{4.29}$$

and hence

$$V_h - V_\phi = M^2(2m_0^2 - A^2/2)\left[\frac{1}{\lambda_3} - \frac{1}{\lambda_1}\right]. \tag{4.30}$$

Hence if $\lambda_3 < \lambda_1$ (so that inflation ends in the h-vacuum) then for $V_h > V_\phi$ we require

$$2m_0^2 > A^2/2. \tag{4.31}$$

It is easy to check from Eq. (4.2) that this is again a sufficient condition that $V''(\pi/2)$ be positive. On the other hand, there is then a range

$$\frac{A^2}{2} < 2m_0^2 < \frac{\lambda_1}{\lambda_3}\frac{A^2}{2} \tag{4.32}$$

for which the h-vacuum is *also* a local minimum. We will see that this scenario is not consistent with a graceful exit from Higgs thermal inflation, and hence for a cosmologically acceptable potential, we must demand

$$2m_0^2 > \frac{\lambda_1}{\lambda_3}\frac{A^2}{2}. \tag{4.33}$$

5 Inflation and reheating

5.1 F-term inflation

We assume that the vevs of MSSM fields apart from the Higgs are negligible, in which case the relevant tree potential is

$$\begin{aligned} V_{\rm tree} = {} & |\lambda_1\phi\overline{\phi} - \lambda_3 h_1 h_2 - M^2|^2 + \left[\lambda_1^2(|\phi|^2 + |\overline{\phi}|^2) + \lambda_3^2(|h_1|^2 + |h_2|^2)\right]|s|^2 \\ & + \frac{1}{2}g'^2\left(q_\Phi(\phi^*\phi - \overline{\phi}^*\overline{\phi}) + q_H(h_1^\dagger h_1 - h_2^\dagger h_2)\right)^2 \\ & + \tfrac{1}{8}g_2^2\sum_a(h_1^\dagger\sigma^a h_1 + h_2^\dagger\sigma^a h_2)^2 + \tfrac{1}{8}g_1^2(h_1^\dagger h_1 - h_2^\dagger h_2)^2 \\ & + V_{\rm soft}. \end{aligned} \tag{5.1}$$

The soft terms in $V_{\rm soft}$ are those appearing in Eqs. (3.2), (3.14), and are all suppressed by at least one power of $m_{\frac{3}{2}}$. The most important soft term for inflation is one linear in s, the effect of which we assume is small compared with the radiative correction. We will see in Eq. (5.7) that this implies tuning below O(1) only if the couplings $\lambda_{1,3}$ are very small. We also assume that the higher order terms in the Kähler potential do not contribute significantly.

At large s, and with all other fields vanishing, the potential is approximately

$$V = M^4 + \Delta V_1, \tag{5.2}$$

where ΔV_1 represents the one-loop corrections, which dominate the soft terms. As S is coupled only to Φ, $\overline{\Phi}$ and $H_{1,2}$, the contribution to the one-loop scalar potential is [32]

$$\begin{aligned} \Delta V_1 = \frac{1}{32\pi^2}\Bigg[& (\lambda_1^2 s^2 + \lambda_1 M^2)^2 \ln\left(\frac{\lambda_1^2 s^2 + \lambda_1 M^2}{\mu^2}\right) + (\lambda_1^2 s^2 - \lambda_1 M^2)^2 \ln\left(\frac{\lambda_1^2 s^2 - \lambda_1 M^2}{\mu^2}\right) \\ & + 2(\lambda_3^2 s^2 + \lambda_3 M^2)^2 \ln\left(\frac{\lambda_3^2 s^2 + \lambda_3 M^2}{\mu^2}\right) + 2(\lambda_3^2 s^2 - \lambda_3 M^2)^2 \ln\left(\frac{\lambda_3^2 s^2 - \lambda_3 M^2}{\mu^2}\right) \\ & - 2\lambda_1^4 s^4 \ln\left(\frac{\lambda_1^2 s^2}{\mu^2}\right) - 4\lambda_3^4 s^4 \ln\left(\frac{\lambda_3^2 s^2}{\mu^2}\right)\Bigg]. \end{aligned} \tag{5.3}$$

For large s (meaning $\lambda_{1,3}s^2 \gg M^2$) the potential can be written as

$$V(s) \simeq M^4 \left[1 + \alpha \ln \frac{2s^2}{s_c^2}\right], \tag{5.4}$$

where an $\mathrm{O}(\alpha)$ correction to M^4 has been dropped, and

$$\alpha = \frac{\lambda^2}{16\pi^2}, \quad \lambda = \sqrt{\lambda_1^2 + 2\lambda_3^2}, \quad s_c^2 = M^2/\lambda. \tag{5.5}$$

We will neglect supergravity contributions in the potential, which will require a small coupling c of the quartic term $c|s|^4/m_{\mathrm{P}}^2$ in the Kahler potential, and impose a constraint [33]

$$\lambda \lesssim 0.06. \tag{5.6}$$

There are also potentially important contributions from the linear soft term $\rho M^2 m_{\frac{3}{2}} s +$ c.c.. These are negligible provided

$$\rho \ll \frac{\lambda^3}{16\pi^2}\frac{s_c}{m_{\frac{3}{2}}}. \tag{5.7}$$

We will shortly see that $s_c \sim 10^{16}$ GeV, so assuming $m_{\frac{3}{2}} \sim 10^5$ GeV, a soft term with $\rho \sim 1$ is negligible provided

$$10^{-3} \lesssim \lambda. \tag{5.8}$$

Henceforth we will assume that the Kähler potential is canonical and that λ is in the range given by Eqs. (5.6), (5.8). We note, however, that interesting consequences for the spectral index flow from a non-canonical Kähler potential [24] and from couplings small enough for the soft term to contribute [25].

5.2 Perturbation amplitudes

The scalar and tensor power spectra $\mathcal{P}_s$, $\mathcal{P}_t$ and the scalar spectral index n_s generated on a scale k equal to the co-moving Hubble scale aH at N_k e-foldings before the end of inflation are given by the standard formulae (see e.g. [34]),

$$\mathcal{P}_s(k) \simeq \frac{1}{24\pi^2}\frac{2N_k}{\alpha}\left(\frac{M}{m_p}\right)^4 = \frac{4N_k}{3}\left(\frac{s_c}{m_p}\right)^4, \tag{5.9}$$

$$\mathcal{P}_t(k) \simeq \frac{1}{6\pi^2}\left(\frac{M}{m_p}\right)^4 = \frac{8}{3}\alpha\left(\frac{s_c}{m_p}\right)^4, \tag{5.10}$$

$$n_s \simeq \left(1 - \frac{1}{N_k}\right). \tag{5.11}$$

The WMAP7 best-fit values for $\mathcal{P}_s(k_0)$ and n_s at a pivot scale $k = k_0 = 0.002\ h\mathrm{Mpc}^{-1}$ in the standard ΛCDM model are [35]

$$\mathcal{P}_s(k_0) = (2.43 \pm 0.11) \times 10^{-9}, \quad n_s = 0.963 \pm 0.012(68\%\mathrm{CL}). \tag{5.12}$$

From this data we infer that

$$\frac{s_c}{m_{\rm P}} \simeq 2.9\times 10^{-3}\left(\frac{27}{N_{k_0}}\right)^{\frac{1}{4}}, \quad N_{k_0} = 27^{+13}_{-7}, \tag{5.13}$$

showing approximately a 2σ discrepancy with the standard Hot Big Bang result $N_{k_0} \simeq 58 + \ln(T_{\rm rh}/10^{15}\ \text{GeV})$ (assuming only MSSM degrees of freedom at $T_{\rm rh}$). We will see shortly that the reheat temperature lies in a range around 10^{14} GeV, and in Section 7 that there are $N_\theta \simeq 15$ e-foldings of thermal inflation at a lower scale. Therefore one can estimate $N_{\rm Fti} \simeq 42(1)$ e-foldings of F-term inflation while the pivot scale k_0 is outside the horizon, where the uncertainty comes from the range of reheat temperatures, given in Eq. (5.19). The scalar spectral index is thereby reduced to

$$n_s \simeq \left(1 - \frac{1}{N_{\rm Fti}}\right) \simeq 0.976(1). \tag{5.14}$$

Lower values of the spectral index are possible if λ drops below the limit (5.8) and the linear soft term comes into play [25].

5.3 End of inflation and reheating

F-term inflation ends when one set of scalar fields becomes unstable. If $\lambda_3 > \lambda_1$, the ϕ, $\overline{\phi}$ pair become unstable first, and inflation ends at the critical value $s_{c1}^2 = M^2/\lambda_1$. The fields ϕ, $\overline{\phi}$ gain vevs and the universe makes a transition to the $\mathrm{U}(1)'$-broken phase described by Eq. (3.3)-Eq. (3.5). On the other hand, if $\lambda_3 < \lambda_1$, the Higgs fields become unstable first, the critical value of s is $s_{c3}^2 = M^2/\lambda_3$, and the universe makes a transition to a phase where h_1 and h_2 develop vevs of order the unification scale rather than $\phi, \overline{\phi}$. In this phase the $SU(2)_L$ symmetry is broken.

At first sight, this would appear to rule out the model with $\lambda_3 < \lambda_1$. However, provided the correct (small Higgs vev) vacuum has the lowest energy density at zero temperature, the universe can seek the true vacuum when thermal corrections become sub-dominant. We will establish in Section 7 that the evolution to the true ground state proceeds by a period of inflation.

Assuming that $\lambda_3 < \lambda_1$, inflation exits to the h-vacuum, with symmetry-breaking

$$\mathrm{SU}(2)\otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)' \quad \to \quad \mathrm{U}(1)_{\rm em}\otimes \mathrm{U}(1)''. \tag{5.15}$$

Here, $\mathrm{U}(1)''$ is generated by the linear combination of hypercharge and $\mathrm{U}(1)'$ generators which leaves the Higgses invariant:

$$Y'' = Y' - (q_L + q_E)Y. \tag{5.16}$$

There are still two Abelian symmetries, and SU(2) is completely broken with no discrete subgroup. Hence cosmic strings are not formed at this transition.

We expect reheating to be very rapid [36–41], as the period of oscillation of the fields is of order M^{-1}, which is much less than a Hubble time, and the couplings of

the Higgs field are not all small. Hence the universe regains a relativistic equation of state almost immediately, and thermalises at a temperature $T_{\rm rh1}$ given by

$$T_{\rm rh1} = \left(\frac{30}{g_{\rm rh1}\pi^2}\right)^{\frac{1}{4}} M = \left(\frac{30}{g_{\rm rh1}\pi^2}\right)^{\frac{1}{4}} \sqrt{\lambda} s_c \tag{5.17}$$

where $g_{\rm rh1}$ is the effective number of relativistic degrees of freedom at temperature $T_{\rm rh1}$. From (5.13), and taking $g_{\rm rh1} = 915/4$ (a slight overestimate), we find

$$T_{\rm rh1} \simeq 2.2\sqrt{\lambda} \times 10^{15}\ \text{GeV}. \tag{5.18}$$

Hence the range of reheat temperatures corresponding to the range of couplings defined by Eq. (5.6) and Eq. (5.8) is

$$0.7 \times 10^{14} \lesssim T_{\rm rh1}/\text{GeV} \lesssim 5 \times 10^{14}. \tag{5.19}$$

Finally, we note that large vevs of other fields along supersymmetric flat directions can lead to blocking of particle production during reheating [27]. On the other hand, radiative corrections during inflation generically generate masses of order y^2H^2 [42], where y is a combination of Yukawa couplings, and so we expect that other vevs besides that of the inflaton will be generally small. We leave a detailed examination of the flat directions for another work, assuming for now that any flat directions which do not have y of order 1 are small.

5.4 High temperature ground state

As the universe reheats, it will seek a minimum of the finite temperature effective potential, or equivalently the free energy density. To discuss the free energy, it is convenient to define a dimensionful field $X = v_+\chi$, with $v_+ = \sqrt{2}v_\phi = 2M/\sqrt{\lambda_1}$. The free energy density can then be expressed as

$$f(X,T) = -\frac{\pi^2}{90} g_{\rm eff}(X,T)T^4, \tag{5.20}$$

where $g_{\rm eff}(X,T)$ is the effective number of relativistic degrees of freedom at temperature T. At weak coupling, $g_{\rm eff}(X,T)$ can be calculated in the high-temperature expansion for a particle of mass $m \ll T$ [43],

$$g_{\rm eff}(X,T) \simeq c_0 - c_1 \frac{90}{\pi^2}\frac{m^2}{T^2}, \tag{5.21}$$

where there are contributions to c_0 of $1, \frac{7}{8}$ and to c_1 of $\frac{1}{24}, \frac{1}{48}$ for bosons and fermions respectively. For particles with $m > T$, $g_{\rm eff}$ is exponentially suppressed.

We can see that $X = 0$ is a local minimum for temperatures $m_{\frac{3}{2}} \ll T \lesssim M$, because away from that point the $\mathrm{U}(1)''$ gauge boson develops a mass proportional to $\langle\phi\rangle$, and so $g_{\rm eff}$ decreases. For similar reasons $X_\phi = v_+\pi/2$ is also a local minimum: away from that point the MSSM particles develop masses and again reduce $g_{\rm eff}$.

In fact, by counting relativistic degrees of freedom at temperatures $m_{\frac{3}{2}} \ll T \lesssim M$ one finds that X_ϕ is the global minimum at high temperature. In the h-vacuum the relativistic species are the $\Phi, \overline{\Phi}$ chiral multiplets and the $\mathrm{U}(1)''$ gauge multiplet. In the ϕ-vacuum, the particles of the MSSM are all light relative to T. Hence

$$f(0,T) \simeq -\frac{15}{2}\frac{\pi^2}{90}T^4, \tag{5.22}$$

$$f(X_\phi,T) \simeq -\frac{915}{4}\frac{\pi^2}{90}T^4. \tag{5.23}$$

The minima of the free energy density are separated by a free energy barrier of height $\sim T^4$. The transition rate can be calculated in the standard way [44] by calculating the free energy of the critical bubble E_c. The transition rate per unit volume is then

$$\Gamma \sim T^4 \left(\frac{E_c}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{E_c}{T}\right). \tag{5.24}$$

The critical bubble is a solution to the equation

$$X'' + \frac{2}{r}X' + V_{\text{eff}}^T(X) = 0, \tag{5.25}$$

where r is the radial distance from the bubble centre, and we have neglected O(1) complications in the kinetic term from the non-linear field transformation. An order-of-magnitude estimate can be given, recognising that X has to change by an amount $\Delta X \sim v_+$ from the inside to the outside of the bubble, while negotiating a local free energy bump of order $\Delta V_{\text{eff}}^T \sim T^4$. Neglecting the damping term, one can translate the equation into a harmonic oscillator problem, finding that the critical bubble radius is approximately

$$r_c \sim \frac{\Delta X}{\sqrt{\Delta V_{\text{eff}}^T}}, \tag{5.26}$$

and so the critical bubble energy

$$E_c \sim \Delta V_{\text{eff}}^T r_c^3 \sim \frac{\Delta X^3}{\sqrt{\Delta V_{\text{eff}}^T}} \sim \frac{v_+^3}{T^2}. \tag{5.27}$$

The universe will stay in the wrong ground state if the transition rate per unit volume is significantly below the Hubble rate per Hubble volume, or $\Gamma < H^4$. Hence the reheat temperature T_{rh1} should be parametrically

$$T_{\text{rh}} < \frac{v_+}{[\ln(m_{\text{P}}/v_+)]^{\frac{1}{3}}}. \tag{5.28}$$

Recalling that $T_{\text{rh}} \simeq M$ and $v_+ = 2M/\sqrt{\lambda_1}$, we see that if inflation exits to the h-vacuum it is likely that the universe continues to evolve with large (inflation-scale) Higgs vevs, provided $\lambda_1 \ll 1$.

6 Review of gravitino constraints

There are strong constraints on the gravitino mass and lifetime from cosmology [9–12]. If the gravitinos are unstable, they can conflict with Big Bang Nucleosynthesis (BBN) by photodissociating light elements, or they can decay directly into the LSP, which in turn produces a limit from the known density of dark matter in the standard cosmological model. The gravitino may also be the LSP, in which case the dark matter constraint applies directly.

Gravitinos are produced by collisions of high-energy particles in the thermal bath, principally gluons and gluinos, with an abundance of approximately [45]

$$Y^{\rm th}_{\frac{3}{2}} \simeq \omega_{\tilde{G}} \left(2.4 + 1.4\frac{M_{\tilde{g}}^2}{m_{\frac{3}{2}}^2}\right) \times 10^{-13} \left(\frac{T_{\rm rh}}{10^9\ \rm GeV}\right), \tag{6.1}$$

where $M_{\tilde{g}}$ is the gaugino mass at the GUT scale. We include an O(1) factor $\omega_{\tilde{G}}$ to take into account the theoretical uncertainties [46–48], arising from the strong dynamics of the coloured plasma.

BBN constraints [45] are not easily summarised, but are much tighter for lighter gravitinos which decay during or after nucleosynthesis, as relevant for the CMSSM. For gravitino masses less than about O(10) TeV, the reheat temperature is bounded above by $T_{\rm rh} \lesssim (0.2-1)\times 10^6$ GeV. For higher gravitino masses, the dark matter density provides a bound, and so it is appropriate use Eq. (6.1) in the limit $M_{\tilde{g}}^2/m_{\frac{3}{2}}^2 \to 0.$.

Given that the LSP density parameter arising from a particular relic abundance in the MSSM is

$$\Omega_{\rm LSP}h^2 \simeq 2.8\times 10^{10}\frac{m_{\rm LSP}}{100\,\rm GeV}Y_{\frac{3}{2}}, \tag{6.2}$$

the LSP density parameter from (high mass) thermally produced gravitinos can be found as

$$\Omega_{\rm LSP}h^2 \simeq \omega_{\tilde{G}} 6\times 10^{-3}\frac{m_{\rm LSP}}{100\ \rm GeV}\left(\frac{T_{\rm rh}}{10^9\ \rm GeV}\right). \tag{6.3}$$

This must be less than or equal to the dark matter abundance inferred from the CMB [35]

$$\Omega_{\rm dm}h^2 \simeq 0.11. \tag{6.4}$$

The presence of cosmic strings in our model, although affecting the CMB power spectrum, does not significantly affect this inferred value [49].

In our model, we will see that the gravitinos generated by the first stage of reheating are diluted by a period of thermal inflation. The constraint therefore applies to reheating after thermal inflation. We will also see that the second reheat temperature is about 10^9 GeV, and so we can only tolerate unstable gravitinos of mass greater than about 10 TeV in order not to spoil BBN. This is natural in AMSB, problematic in GMSB, while the CMSSM keeps $m_{\frac{3}{2}}$ as a separate parameter.

There are also non-thermal production mechanisms from coherent oscillations of the inflaton [50, 51] and from ordinary perturbative decay [52], whose rates depend on the inflaton mass and vev. We will see in the next section that the relevant inflaton

mass and vev will be those of the Higgs. However the BBN constraints mean that the gravitino, when it is not the LSP, must be much more massive than the Higgs and so cannot be produced by direct decays. Hence only thermal production is relevant.

7 Higgs thermal inflation and gravitinos

As the temperature falls, the energy density difference between the vacua becomes comparable to thermal energy density, and the universe can seek its true ground state, which is $\chi = \pi/2$, the ϕ-vacuum.

At zero temperature we can write the difference in energy density between the h-vacuum and the ϕ-vacuum as (see Eqs. (4.17), (A.13), (4.26) and (4.30))

$$\Delta V_{\rm eff}^0 \simeq v_+^2 m_{\rm sb}^2 \tag{7.1}$$

where we recall that $v_+^2 = 4M^2/\lambda_1$, and we have defined an effective SUSY-breaking scale $m_{\rm sb}$. In the supersymmetry-breaking scenarios under consideration

$$m_{\rm sb}^2 \simeq \begin{cases} m_{\frac{3}{2}}^2 \left(\frac{g'^2}{16\pi^2}\right)^2 q_\phi^2 Q, & \text{(AMSB)}, \\ \Lambda_{\rm s}^2 \frac{\lambda_1}{\lambda_3} \left[\frac{3}{8}\left(\frac{g_2^2}{16\pi^2}\right)^2 + \frac{1}{8}\left(\frac{g_1^2}{16\pi^2}\right)^2\right] & \text{(GMSB)}, \\ \frac{1}{2}(m_0^2 - A^2/4)\left[\frac{\lambda_1}{\lambda_3} - 1\right] & \text{(CMSSM)}. \end{cases} \tag{7.2}$$

A period of thermal inflation [53] starts at

$$T_{\rm i} \simeq \left(\frac{30}{g_{\rm i}\pi^2} v_+^2 m_{\rm sb}^2\right)^{\frac{1}{4}}, \tag{7.3}$$

where $g_{\rm i}$ is the effective number of degrees of freedom at temperature $T_{\rm i}$. The CMB normalisation (5.13) for N e-foldings of standard hybrid inflation gives $(v_+/m_{\rm P}) \simeq 5\times 10^{-3}(40/N)^{\frac{1}{4}}$. Using the number of degrees of freedom for a $\mathrm{U}(1)_{\rm em}\otimes \mathrm{U}(1)''$ theory with two light chiral multiplets Φ and $\overline{\Phi}$, $g_{\rm i} = 15$, we have (on dropping the unimportant dependence on N)

$$T_{\rm i} \simeq 2.2\times 10^9 \left(\frac{m_{\rm sb}}{1~\text{TeV}}\right)^{\frac{1}{2}}~\text{GeV}. \tag{7.4}$$

The h-vacuum must be a local maximum at zero temperature, i.e. the soft mass terms $m_\phi^2|\phi|^2 + m_{\bar\phi}^2|\overline{\phi}|^2$ must be negative. If the h-vacuum were a local minimum, one can estimate that the tunnelling rate per Hubble time per Hubble volume [54] would be

$$\frac{\Gamma}{H^4} \sim \frac{m_{\rm sb}^4 m_{\rm P}^4}{M^8} e^{-S_E}, \tag{7.5}$$

where S_E is the action of the Euclidean tunnelling solution. This ratio must be of order unity for the universe not to remain trapped in the false vacuum [55], and since the prefactor is much less than unity, we see that we cannot allow a metastable h-vacuum for a graceful exit from thermal inflation.

Thermal inflation continues until the quadratic term in the thermal potential $g'^2T^2(|\phi|^2+|\overline{\phi}|^2)$ becomes the same size as the negative soft mass terms. Near the false vacuum, the high temperature effective potential for the field X breaking the $\mathrm{U}(1)''$ symmetry can be written [44, 56]

$$V_{\text{eff}}(X) \simeq \tfrac{1}{2}\gamma(T^2-T_0^2)X^2 - \tfrac{1}{3}\delta T X^3 + \tfrac{1}{4}\lambda_X X^4, \tag{7.6}$$

where γ, δ and λ_X are dimensionless constants, and $T_0 \simeq |m_\phi|/g'$. The cubic term arises from the gauge boson, and the transition is first order provided $\lambda_X < e^4$, where e is the effective $\mathrm{U}(1)''$ gauge coupling [44, 56].

Hence the transition which ends thermal inflation takes place at $T_{\mathrm{e}} \sim m_{\mathrm{sb}}$, and the number of e-foldings of thermal inflation is

$$N_\theta \simeq \tfrac{1}{2}\ln\left(\tfrac{v_+}{m_{\mathrm{sb}}}\right) \simeq 15 - \ln\left(\tfrac{m_{\mathrm{sb}}}{1\,\mathrm{TeV}}\right), \tag{7.7}$$

Thus gravitinos will be diluted to unobservably low densities, as will any baryon number generated prior to thermal inflation, and any other dangerous GUT-scale relics such as monopoles.

After thermal inflation ends, there is another period of reheating as the energy of the modulus X is converted to particles. Around the true vacuum, the X is mostly Higgs, and so its large amplitude oscillations will be quickly converted into the particles of the MSSM. The natural oscillation frequency around $\chi = \pi/2$ is of order $m_{\frac{3}{2}}$, while the Hubble rate is of order $m_{\frac{3}{2}}M/m_{\mathrm{P}}$. Hence in much less than an expansion time, the vacuum energy will be efficiently converted into thermal energy. The reheat temperature following thermal inflation is thus

$$T_{\mathrm{rh2}} = \left(\frac{30}{g_{\mathrm{rh2}}\pi^2}\Delta V_{\mathrm{eff}}^0\right)^{\frac{1}{4}} \simeq 0.5T_{\mathrm{i}} \simeq 1.1\times 10^9 \left(\frac{m_{\mathrm{sb}}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}}\ \mathrm{GeV}, \tag{7.8}$$

where g_{rh2} is the effective number of relativistic degrees of freedom at T_{rh2}, given its MSSM value $g_{\mathrm{rh2}} = 915/4$. This second reheating regenerates the gravitinos, and we may apply the gravitino density formula Eq. (6.3), finding

$$\Omega_{\mathrm{LSP}}h^2 \simeq 6\times 10^{-3}\omega_{\tilde{G}}\frac{m_{\mathrm{LSP}}}{100\ \mathrm{GeV}}\left(\frac{m_{\mathrm{sb}}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}} \tag{7.9}$$

We can convert the relic density into a constraint on the effective SUSY-breaking scale m_{sb}, requiring that the LSP density is less than or equal to the observed dark matter abundance, $\Omega_{\mathrm{dm}}h^2 \simeq 0.11$.

$$m_{\mathrm{sb}} \lesssim 3\times 10^2\frac{1}{\omega_{\tilde{G}}^2}\left(\frac{m_{\mathrm{LSP}}}{100\ \mathrm{GeV}}\right)^{-2}\ \mathrm{TeV} \tag{7.10}$$

The parameter m_{sb} is directly related to physical observables differently in the different SUSY-breaking schemes, for which we can derive constraints.

7.1 Gravitino constraint in AMSB

Using Eq. (7.10) and the expression for $m_{\rm sb}$ in Eq. (7.2), we find

$$m_{\frac{3}{2}} \lesssim \frac{5\times 10^4}{g'^2 q_\Phi \sqrt{Q}} \left(\omega_{\tilde G}\frac{m_{\rm LSP}}{100\ {\rm GeV}}\right)^{-2}\ {\rm TeV}. \tag{7.11}$$

Hence AMSB-based models requires a high gravitino mass in order to saturate the bound and generate the dark matter.

We can be a bit more precise if we use use the phenomenological relations derived in [13]. Firstly, in order to fit μ_h we have (using Eqs. (3.13), (4.7))

$$q_\Phi^2 g'^2 \simeq \frac{\lambda_1}{\lambda_3}, \tag{7.12}$$

while we can use a phenomenological formula for the LSP mass

$$m_{\rm LSP} \simeq 3.3\times 10^{-3} m_{\frac{3}{2}}. \tag{7.13}$$

Hence

$$m_{\frac{3}{2}} \lesssim 360 \left(\frac{1}{\omega_{\tilde G}^2}\frac{q_\Phi}{\sqrt{Q}}\frac{\lambda_3}{\lambda_1}\right)^{\frac{1}{3}}\ {\rm TeV}. \tag{7.14}$$

We also have a constraint (Eq. (4.21)) on λ_3/λ_1 from requiring the exit to a false h-vacuum. Hence in order for the LSP in this model to comprise all the dark matter, we have

$$\left(\frac{1}{\omega_{\tilde G}^2}\frac{q_\Phi}{\sqrt{Q}}\frac{q_H^2}{q_\Phi^2}\right)^{\frac{1}{3}} \lesssim \frac{m_{\frac{3}{2}}}{360\ {\rm TeV}} \lesssim \left(\frac{1}{\omega_{\tilde G}^2}\frac{q_\Phi}{\sqrt{Q}}\right)^{\frac{1}{3}}. \tag{7.15}$$

For example, taking $q_L = 0$ as in [13], we find that $m_{\frac{3}{2}}$ is independent of q_E and in the range

$$130\,\omega_{\tilde G}^{-\frac{2}{3}}\ {\rm TeV} \lesssim m_{\frac{3}{2}} \lesssim 250\,\omega_{\tilde G}^{-\frac{2}{3}}\ {\rm TeV}, \tag{7.16}$$

where we recall from the discussion around Eq. (6.1) that $\omega_{\tilde G}$ is O(1). It was noted in [13] that a Higgs of mass 125 GeV demands a gravitino mass of about 140 TeV in sAMSB, which is compatible with an LSP produced by gravitino decays being the dark matter.

7.2 Gravitino constraint in GMSB

In the GMSB framework the LSP is usually the gravitino, whose mass is given by

$$m_{\frac{3}{2}} = \frac{1}{\sqrt{3}kN_{\rm mi}}\left(\frac{M_X}{m_{\rm P}}\right)\Lambda_{\rm g}, \tag{7.17}$$

where $k<1$ parametrises the fraction of the total F-term contained in the messenger sector, and we recall that M_X is the messenger scale. It is more convenient to phrase the dark matter constraint in terms of the larger electroweak gaugino mass M_2,

which dominates in the equation for the SUSY-breaking scale $\Lambda_{\rm g}$, as in Eq. (4.23), and therefore Eq. (7.2) can be rewritten

$$m_{\rm sb}^2 \simeq \frac{3}{8}\frac{\lambda_1}{\lambda_3}\frac{M_2^2}{N_{\rm mi}}. \tag{7.18}$$

Hence

$$\left(\frac{m_{\frac{3}{2}}}{1\ {\rm TeV}}\right)^2 \lesssim 5\sqrt{N_{\rm mi}}\sqrt{\frac{\lambda_3}{\lambda_1}}\frac{1}{\omega_{\tilde{G}}^2}\left(\frac{M_2}{1\ {\rm TeV}}\right)^{-1}. \tag{7.19}$$

The bound can be saturated for a TeV-scale gravitino with TeV-scale gaugino masses without special tuning of the ratio λ_3/λ_1. A lighter gravitino forces the gaugino mass upwards.

There is a separate constraint from decays of the NLSP (which is generally a neutralino for unless M_X is small), which may interfere with Big Bang Nucleosynthesis (see e.g. [31]). A careful analysis of the nucleosynthesis constraints [57] shows that a messenger mass of up to about 10^{14} GeV is allowed, before hadronic jets injected after 10^4 s results in the overproduction of ^{7}Li. The combination of the dark matter and BBN constraints $M_X \lesssim 10^{14}$ GeV may be written

$$\left(\frac{m_{\frac{3}{2}}}{1\ {\rm TeV}}\right)^3 \lesssim 0.5\sqrt{N_{\rm mi}}\sqrt{\frac{\lambda_3}{\lambda_1}}\frac{1}{\omega_{\tilde{G}}^2}. \tag{7.20}$$

7.3 Gravitino constraint in the CMSSM

In the CMSSM, the gravitino bound Eq. (7.10) can be expressed in terms of the soft scalar masses m_0 and the trilinear parameter A from Eq. (7.2), as

$$\left(m_0^2 - A^2/4\right)^{\frac{1}{2}} \lesssim 5\times 10^2\left(\frac{\lambda_1}{\lambda_3}-1\right)^{-\frac{1}{2}}\left(\omega_{\tilde{G}}\frac{m_{\rm LSP}}{100\ {\rm GeV}}\right)^{-2}\ {\rm TeV}. \tag{7.21}$$

This is a very weak bound, unless the ratio λ_3/λ_1 is very small: hence there is generically a very low density of LSP dark matter generated by decays of gravitinos. Instead, the CMSSM can generate an acceptable dark matter density through the standard freeze-out scenario [58] (see [59–61] for recent analyses of the the CMSSM parameter space in the light of recent Higgs results). This requires that the gravitino mass is larger than about 10 TeV to avoid BBN constraints [45]. We conclude that the Higgs thermal inflation solution generally has no effect on the gravitino problem in the CMSSM, beyond determining the reheat temperature and hence the standard BBN-induced lower bound gravitino mass.

8 Conclusions

In this paper we have shown how models which couple F-term hybrid inflation with the MSSM without extra ingredients naturally realise a period of thermal inflation, with a reheat temperature of around 10^9 GeV, while generating the Higgs μ-term. The

inflation is driven by the relaxation of the Higgs fields to zero in a potential generated by the Higgs and waterfall field soft terms. This second period of intermediate scale inflation, which we have called Higgs thermal inflation, has a number of beneficial effects. It solves the gravitino overabundance problem of supersymmetric cosmology, while still maintaining the possibility of leptogenesis. It reduces the cosmic string mass per unit length so that CMB bounds are satisfied, and renders it independent of the inflaton couplings. Hence the scalar spectral index is not driven to unity in the effort to make the strings light, from which the tight constraints on standard F-term hybrid inflation are generated [8]. The period of thermal inflation means a reduced number of e-foldings of F-term inflation are required, and the scalar spectral index is reduced: in the range of inflaton couplings where the inflaton potential is dominated by the radiative corrections, we find $n_s \simeq 0.976(1)$.

The MHISSM is the simplest, attractive, formulation of this scenario, and generates right-handed neutrino masses as well as the Higgs μ-term. We found constraints on the couplings and soft terms in order for the scenario to work: i.e. for F-term inflation to exit towards a vacuum with inflation-scale vevs for the Higgs field, and for that vacuum to be unstable. We investigated the implications of these constraints in three popular supersymmetry-breaking scenarios: AMSB (where the model coincides with strictly anomaly-mediated supersymmetry-breaking [13]), GMSB, and the CMSSM. We found constraints on the ratio of the inflaton couplings in AMSB (Eq. (4.18)) and GMSB (Eq. (4.27)), and that in the CMSSM the soft scalar mass must be greater than the half the magnitude of the soft trilinear term, multiplied by the square root of the ratio of the inflaton couplings (Eq. (4.33)).

In AMSB, the gravitino problem becomes the gravitino solution: the observed dark matter density can be generated by the decays of gravitinos which are produced thermally following Higgs thermal inflation. In GMSB, the gravitino is the LSP, and a weak upper bound on its mass of about 1 TeV follows from the combined requirement that it supply the dark matter without NLSP decays spoiling nucleosynthesis. In the CMSSM, the density of thermally-produced gravitinos is generally sub-dominant, and the standard freeze-out scenario must do the work of making neutralino dark matter. However, the gravitinos must decay early enough not to spoil nucleosynthesis, meaning that the gravitino mass must be O(10) TeV or greater.

A reheat temperature of 10^9 GeV is broadly consistent with thermal leptogenesis, provided at least one right-handed neutrino is light enough to be thermally produced. We leave the details for a future publication.

The MHISSM predicts the formation cosmic strings, with dimensionless mass per unit length estimated as $G\mu_\mathrm{s} \simeq 10^{-7}$ [13]. While satisfying current CMB bounds, there are tight bounds on the GeV-scale cosmic γ-ray spectrum [62], so strings should have a very small branching fraction into γ. Strings may instead decay into gravitational waves, but there are also increasingly strict bounds on the stochastic gravitational wave background from pulsar timing [63, 64]. Should the bounds ultimately fall below the predicted value of $G\mu_\mathrm{s}$, this will rule out the MHISSM but not the Higgs thermal inflation scenario in general, which remains a possibility whenever the inflaton is coupled to a set of waterfall fields which include the Higgs.

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A sAMSB soft parameters and inflaton coupling constraints

In this appendix we give formulae for a more accurate calculation of the sAMSB soft parameters in Section 4.1, relevant for the calculation of the vacuum energies and hence the constraint on λ_3/λ_1.

The h_{λ_1} and h_{λ_3} trilinear soft terms are determined in accordance with Eq. (4.4):

$$h_{\lambda_1} = -m_{\frac{3}{2}}\frac{\lambda_1}{16\pi^2}\left(3\lambda_1^2 + \frac{1}{2}\operatorname{Tr}\lambda_2^2 + 2\lambda_3^2 - 4q_\Phi^2 g'^2\right), \tag{A.1}$$

$$\begin{aligned} h_{\lambda_3} = &-m_{\frac{3}{2}}\frac{\lambda_3}{16\pi^2}(\operatorname{Tr}Y_E Y_E^\dagger + 3\operatorname{Tr}Y_D Y_D^\dagger + 3\operatorname{Tr}Y_U Y_U^\dagger + \lambda_1^2 + 4\operatorname{Tr}\lambda_3^2 \\ &-3g_2^2 - g_1^2 - 4q_H^2 g'^2), \end{aligned} \tag{A.2}$$

while the mass soft terms are

$$m_\phi^2 = m_{\frac{3}{2}}^2\frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(\lambda_1^2 + \tfrac{1}{2}\operatorname{Tr}\lambda_2^2 - 2g'^2 q_\Phi^2\right), \tag{A.3}$$

$$m_{\bar\phi}^2 = m_{\frac{3}{2}}^2\frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(\lambda_1^2 - 2g'^2 q_\Phi^2\right), \tag{A.4}$$

$$m_{h_1}^2 = m_{\frac{3}{2}}^2\frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(\lambda_3^2 + \operatorname{Tr}Y_E Y_E^\dagger + 3\operatorname{Tr}Y_D Y_D^\dagger - 2g'^2 q_H^2 - \tfrac{1}{2}g_1^2 - \tfrac{3}{2}g_2^2\right), \tag{A.5}$$

$$m_{h_2}^2 = m_{\frac{3}{2}}^2\frac{1}{32\pi^2}\mu\frac{d}{d\mu}\left(\lambda_3^2 + \operatorname{Tr}Y_N Y_N^\dagger + 3\operatorname{Tr}Y_U Y_U^\dagger - 2g'^2 q_H^2 - \tfrac{1}{2}g_1^2 - \tfrac{3}{2}g_2^2\right). \tag{A.6}$$

In the following, we include the SM gauge couplings and the top Yukawa coupling, which we shall denote y_t. We also retain the neutrino Yukawas Y_N, since their magnitude is model dependent. We will assume that the value of $\tan\beta$ is such that it is a good approximation to neglect all the other Yukawas, and we will also neglect λ_1 and λ_3.

The relevant soft breaking parameters are then

$$h_{\lambda_1} = m_{\frac{3}{2}}\frac{\lambda_1}{16\pi^2}\left(4q_\Phi^2 g'^2 - \frac{1}{2}\,\mathrm{Tr}\,\lambda_2^2\right), \tag{A.7}$$

$$h_{\lambda_3} = m_{\frac{3}{2}}\frac{\lambda_3}{16\pi^2}\left(4q_H^2 g'^2 + 3g_2^2 + g_1^2 - 3y_t^2 - \mathrm{Tr}\,Y_N^2\right), \tag{A.8}$$

$$\begin{aligned} m_\phi^2 = -\frac{m_{\frac{3}{2}}^2}{(16\pi^2)^2}\Big[&2q_\Phi^2 Q g'^4 - \mathrm{Tr}\,\lambda_2^4 \\ &- \tfrac{1}{2}\,\mathrm{Tr}\,\lambda_2^2\left\{\tfrac{1}{2}\,\mathrm{Tr}\,\lambda_2^2 + 4\,\mathrm{Tr}\,Y_N Y_N^\dagger - 2(q_\Phi^2 + 2q_N^2)g'^2\right\}\Big], \end{aligned} \tag{A.9}$$

$$m_{\bar\phi}^2 = -\frac{m_{\frac{3}{2}}^2}{(16\pi^2)^2}\left[2q_\Phi^2 Q g'^4\right], \tag{A.10}$$

$$m_{h_1}^2 = -\frac{m_{\frac{3}{2}}^2}{(16\pi^2)^2}\left[2q_H^2 Q g'^4 + \frac{11}{2}g_1^4 + \frac{3}{2}g_2^4\right], \tag{A.11}$$

$$\begin{aligned} m_{h_2}^2 = -\frac{m_{\frac{3}{2}}^2}{(16\pi^2)^2}\Big[&2q_H^2 Q g'^4 + \frac{11}{2}g_1^4 + \frac{3}{2}g_2^4 - 6\,\mathrm{Tr}\,Y_N^4 - 2\,\mathrm{Tr}\,Y_N^2\lambda_2^2 \\ &-2\,\mathrm{Tr}\,Y_N^2\left\{\mathrm{Tr}\,Y_N^2 + 3y_t^2 - 3g_2^2 - g_1^2 - 2(q_N^2 + q_L^2 + q_H^2)g'^2\right\} \\ &-3y_t^2\left(6y_t^2 + \mathrm{Tr}\,Y_N^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2 - 2(q_Q^2 + q_{t^c}^2 + q_H^2)g'^2\right)\Big]. \end{aligned} \tag{A.12}$$

We have assumed above for simplicity that, like λ_2, Y_N is real and diagonal.

The difference in the energies between the large-Higgs and large-ϕ vacua can be written

$$\begin{aligned} \frac{(16\pi^2)^2\lambda_1}{m_{\frac{3}{2}}^2 M^2}(V_h - V_\phi) \simeq\ & 4Q q_\Phi^2 g'^4 + \Delta_{m_\phi^2} + \tfrac{1}{2}\left(4q_\Phi^2 g'^2 + \Delta_{h_1}\right)^2 \\ &-\frac{\lambda_1}{\lambda_3}\left[4Q q_H^2 g'^4 + \Delta_{m_h^2} + \tfrac{1}{2}\left(4q_H^2 g'^2 + \Delta_{h_3}\right)^2\right], \end{aligned} \tag{A.13}$$

where

$$\Delta_{m_\phi^2} = -\tfrac{1}{2}\,\mathrm{Tr}\,\lambda_2^2\left(\tfrac{1}{2}\,\mathrm{Tr}\,\lambda_2^2 + 4\,\mathrm{Tr}\,Y_N Y_N^\dagger - 2(q_\Phi^2 + 2q_N^2)g'^2\right) - \mathrm{Tr}\,\lambda_2^4 \tag{A.14}$$

$$\Delta_{h_1} = -\tfrac{1}{2}\,\mathrm{Tr}\,\lambda_2^2 \tag{A.15}$$

$$\begin{aligned} \Delta_{m_h^2} =\ & 11g_1^4 + 3g_2^4 - 6\,\mathrm{Tr}\,Y_N^4 - 2\,\mathrm{Tr}\,Y_N^2\lambda_2^2 \\ &-2\ \mathrm{Tr}\,Y_N^2\left\{\mathrm{Tr}\,Y_N^2 + 3y_t^2 - 3g_2^2 - g_1^2 - 2(q_N^2 + q_L^2 + q_H^2)g'^2\right\} \\ &-\ 3y_t^2\left(6y_t^2 - \frac{11}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2 - 2(q_Q^2 + q_{t^c}^2 + q_H^2)g'^2\right) \end{aligned} \tag{A.16}$$

$$\Delta_{h_3} = g_1^2 + 3g_2^2 - 3|y_t|^2 - \mathrm{Tr}\,Y_N^2 \tag{A.17}$$

The condition on the couplings deriving from the vacuum energies can therefore be written

$$\frac{\lambda_3}{\lambda_1} > \frac{q_H^2}{q_\phi^2} \frac{1 + (q_H^2/Q)\left(\beta_H^2 \Delta_{m_h^2} + 2\left(1 + \frac{1}{2}\beta_H \Delta_{h_3}\right)^2\right)}{1 + (q_\phi^2/Q)\left(\beta_\phi^2 \Delta_{m_\phi^2} + 2\left(1 + \frac{1}{2}\beta_\phi \Delta_{h_1}\right)^2\right)}, \tag{A.18}$$

where

$$\beta_H = \frac{1}{2q_H^2 g'^2}, \quad \beta_\phi = \frac{1}{2q_\phi^2 g'^2}. \tag{A.19}$$

Taking the values of the SM couplings at the U(1)′ breaking scale to be the values at gauge coupling unification, we find using the renormalisation group analysis of Ref. [13]

$$g_1 \simeq 0.55, \quad g_2 \simeq 0.71, \quad g_3 \simeq 0.70; \quad y_t \simeq 0.51 \tag{A.20}$$

(where we have taken $\tan\beta = 16$). Hence, at this level of accuracy,

$$\begin{aligned} \Delta_{m_h^2} &\simeq 3.5 - 6\,\mathrm{Tr}\,Y_N^4 - 2\,\mathrm{Tr}\,Y_N^2 \lambda_2^2 \\ &\quad -2\,\mathrm{Tr}\,Y_N^2 \left\{\mathrm{Tr}\,Y_N^2 - 1.0 - 2(q_N^2 + q_L^2 + q_H^2) g'^2\right\} \\ &\quad + 1.6\left(q_Q^2 + q_{t^c}^2 + q_H^2\right) g'^2 \qquad (A.21) \\ \Delta_{h_3} &\simeq 1.0 - \mathrm{Tr}\,Y_N^2. \qquad (A.22) \end{aligned}$$

We can derive successive approximations. Firstly, neglecting terms of order q_H^2/Q and q_ϕ^2/Q, which is a good approximation given Eq. (4.14), we have

$$\frac{\lambda_3}{\lambda_1} > \frac{q_H^2}{q_\phi^2}. \tag{A.23}$$

Secondly, we can neglect terms of order $\beta_{H,\phi}$ and higher (which is not necessarily a good approximation), to obtain

$$\frac{\lambda_3}{\lambda_1} > \frac{q_H^2}{q_\phi^2} \frac{1 + 2(q_H^2/Q)}{1 + 2(q_\phi^2/Q)}. \tag{A.24}$$

In the case $q_L = 0$, we obtain Eq. (4.20).

We can also expand in powers of $\beta_{H,\phi}$ while still neglecting terms of order Y_N^2, and bearing in mind that an acceptable electroweak vacuum requires $\mathrm{Tr}\,\lambda_2^2 \simeq 4\beta_H$ [13]), we find to second order

$$\frac{\lambda_3}{\lambda_1} > \frac{q_H^2}{q_\phi^2} \frac{1 + 2(q_H^2/Q)\left(1 + [1.4 + 0.4(q_\phi^2 + q_N^2)/q_H^2]\beta_H + 2.0\beta_H^2\right)}{1 + 2(q_\phi^2/Q)\left(1 + \frac{1}{4}\,\mathrm{Tr}\,\lambda_2^2[1 + 2q_N^2/q_\phi^2] - \frac{1}{2}\beta_\phi\,\mathrm{Tr}\,\lambda_2^2\right)}. \tag{A.25}$$

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